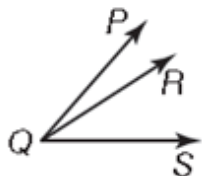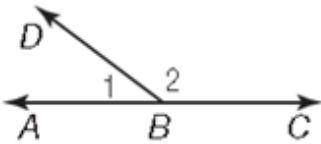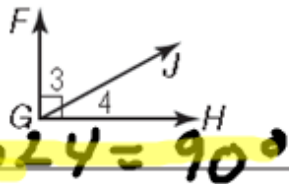# Lesson 2-8
Proving Angle Relationships

You will be able to prove theorems involving supplementary, complementary, congruent and right angles.

Supplementary and Complementary Angles

Protractor Postulate	Given any angle, the measure can be put into one-to-one correspondance with real numbers between 0 and 180.	
Angle Addition Postulate	$m\angle PQR + m\angle RQS = m\angle PQS$	

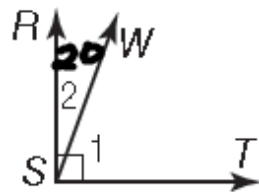| Supplement Theorem	If $\angle 1$ and $\angle 2$ form a linear pair supplementary $\longrightarrow$ $m\angle 1 + m\angle 2 = 180$	
Complement Theorem	If 2 $\angle$'s together form a Rt $\angle$, then they are Comp. $m\angle 3 + m\angle 4 = 90°$	

Example 1   If $\angle 1$ and $\angle 2$ form a linear pair and $m\angle 2 = \underline{115}$, find $m\angle 1$.

$115°$ Q

$2 / 1$

M N P
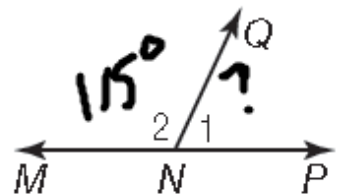
1) $m\angle 1 + m\angle 2 = 180$ Suppl. Thm

2) $m\angle 1 + 115 = 180$ Subst.

$m\angle 1 = 65$ Subt. Prop

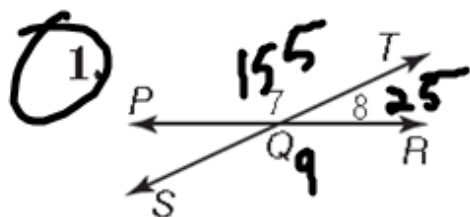**Example 2**   If $\angle 1$ and $\angle 2$ form a right angle and $m\angle 2 = 20$, find $m\angle 1$.

R 20 W

2

S 1 T

$m\angle 1 + m\angle 2 = 90$ Comp Thm

$m\angle 1 + 20 = 90$ Subst.

$m\angle 1 = 70$ Sub Prop

Exercises

Find the measure of each numbered angle and name the theorem that justifies your work.

1. [handwritten: 155, 25, 9]

P, T, 7, 8, Q, R, S

$m\angle 7 = 5x + 5,$
$m\angle 8 = x - 5$

2. [handwritten: 60°, 30, 60°, 3n°]

X, Y, Z, 7, 6, 8, 5, U, V, W

$m\angle 5 = 5x,\ m\angle 6 = 4x + 6,$
$m\angle 7 = 10x,$
$m\angle 8 = 12x - 12$

3. [handwritten: 88, 92, 88]

A, H, 11, 13, J, 12, F, C

[handwritten]
$m\angle 11 = 11x,$ (8)
$+ m\angle 13 = 108 + 12$
$\overline{180}$ **Suppl Thm**
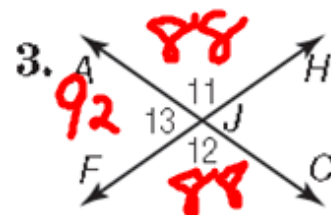
$10x + 12 + 11x = 180$

$21x + 12 = 180$

$\quad\quad -12 \quad -12$
$\overline{\hphantom{21x+12=180}}$
$21x = 168$
$\overline{21} \quad \overline{21}$
$x = 8$

Congruent and Right Angles

Congruent and Right Angles The Reflexive Property of Congruence, Symmetric Property of Congruence, and Transitive Property of Congruence all hold true for angles. The following theorems also hold true for angles.

✱Congruent Supplements Theorem	Angles supplement to the same angle or congruent angles are congruent.
✱Congruent Compliments Theorem	Angles compliment to the same angle or to congruent angles are congruent.
Vertical Angles Theorem	If two angles are vertical angles, then they are congruent.
Theorem 2.9	Perpendicular lines intersect to form four right angles.
Theorem 2.10	All right angles are congruent.
Theorem 2.11	Perpendicular lines form congruent adjacent angles.
Theorem 2.12	If two angles are congruent and supplementary, then each angle is a right angle.
Theorem 2.13	If two congruent angles form a linear pair, then they are right angles.

Write a two-column proof.

Given: $\angle ABC$ and $\angle CBD$ are complementary.

$\angle DBE$ and $\angle CBD$ form a right angle.

Prove: $\angle ABC \cong \angle DBE$
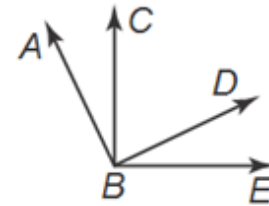
Statements	Reasons
1. $\angle ABC$ and $\angle CBD$ are complementary. $\angle DBE$ and $\angle CBD$ form a right angle.	1. Given
2. $\angle DBE$ and $\angle CBD$ are complementary.	2. Comp Thm
3. $\angle ABC \cong \angle DBE$	3. $\cong$ Comp Thm.

Exercises

Complete each proof.

1. Given: $\overline{AB} \perp \overline{BC}$; ∠1 and ∠3 are complementary. *(marked with red ✗)*
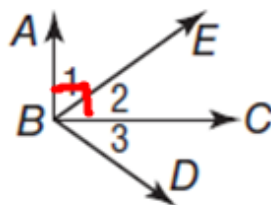
Prove: $\angle 2 \cong \angle 3$

Proof:

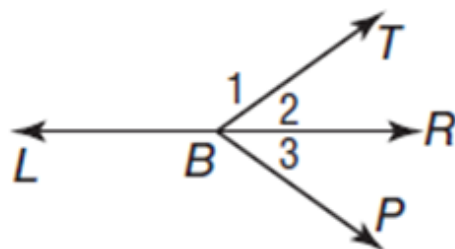| Statements	Reasons
a. $\overline{AB} \perp \overline{BC}$	**a.** Given
b. ∠ABC is a Rt∠	**b.** Definition of ⊥
c. $m\angle ABC = 90$	**c.** Def. of right angle
d. $m\angle ABC = m\angle 1 + m\angle 2$	**d.** Angle Add Post
e. $90 = m\angle 1 + m\angle 2$	**e.** Substitution
f. ∠1 and ∠2 are compl.	**f.** Comp Thm
g. ∠1 and ∠3 are comp	**g.** Given
h. $\angle 2 \cong \angle 3$	**h.** ≅ Comp Thm.

Exercises, cont.

2. Given: $\angle 1$ and $\angle 2$
form a linear pair.
$m\angle 1 + m\angle 3 = 180$

Prove: $\angle 2 \cong \angle 3$

Proof:

Statements	Reasons
a. $\angle 1$ and $\angle 2$ form a linear pair. $m\angle 1 + m\angle 3 = 180$	**a.** Given
b. $m\angle 1 + m\angle 2 = 180$ *or $\angle 1$ & $\angle 2$ are suppl.*	**b.** Suppl. Theorem
c. $\angle 1$ is suppl. to $\angle 3$.	**c.** Suppl Thm.
d. $\angle 2 \cong \angle 3$	**d.** $\angle$s suppl. to the same $\angle$ or $\cong$ $\angle$s are $\cong$. $\cong$ Suppl Thm.