Lessons 2-7
Proving Segment Relationships

CCSS

You will prove theorems involving segment addition and congruence.

Theorems, Postulates, Axioms, *Definitions*

statements that have been proven or accepted to be true .

Two-column proof

a formal proof that organizes *statements* and *reasons* in two columns

# 2-7 Proving Segment Relationships

Segment Addition Two basic postulates for working with segments and lengths are the Ruler Postulate, which establishes number lines, and the Segment Addition Postulate, which describes what it means for one point to be between two other points.

Ruler Postulate	The points on any line or line segment can be put into one-to-one correspondence with real numbers.
Segment Addition Postulate	If A, B, and C are collinear, then point B is between A and C if and only if AB + BC = AC.

$$AB + BC = AC$$
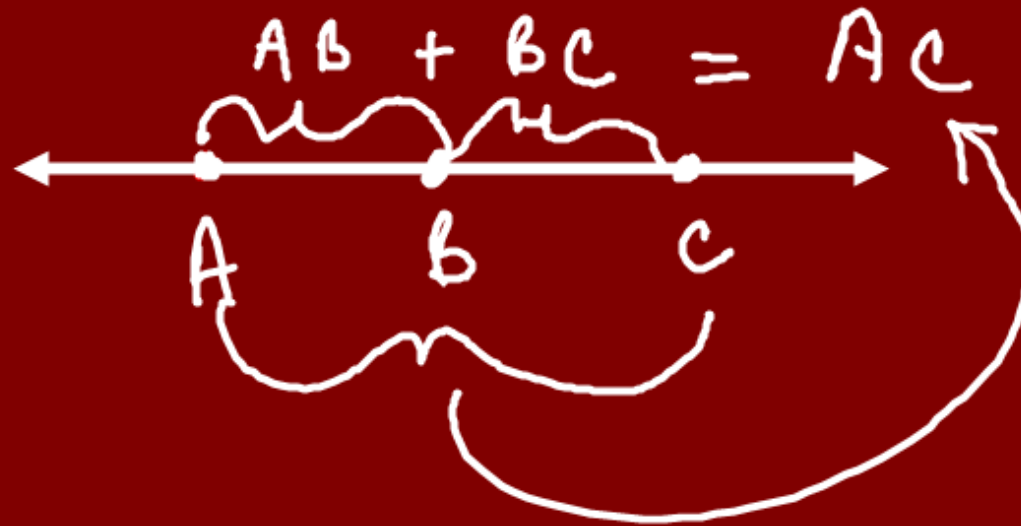
Example

Write a two-column proof.

Given: Q is the midpoint of $\overline{PR}$. → $PQ = QR$
R is the midpoint of $\overline{QS}$. → $QR = RS$
Prove: $PR = QS$
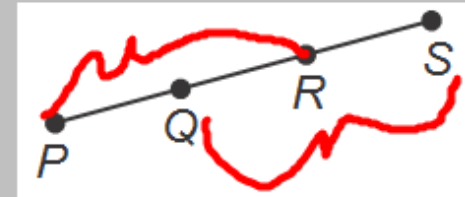
Proof:

Statements	Reasons
1. Q is the midpoint of $\overline{PR}$	1. $\Big\}$ 6. Jiven
2. R is the midpoint of $\overline{QS}$	2. $\Big\}$
3. $PQ = QR$	3. $\Big\}$ Dfn of midpoint
4. $QR = RS$	4. $\Big\}$
5. $PQ + QR = QR + RS$	5. Add Prop
6. $PQ + QR = PR$, $QR + RS = QS$	6. Seg Add Post.
7. $PR = QS$	7. Subst.

Complete each proof.

1. **Given:** $BC = DE$

   *****Prove:** $AB + DE = AC$
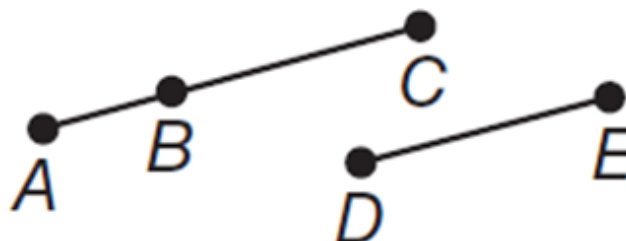
   Proof:

Statements	Reasons
1. $BC = DE$	1. Given
2. $AB + BC = AC$	2. Seg. Add. Post.
*3. $AB + DE = AC$	3. Subst.

2. Given: Q is between [Seg Add Post.] P and R, R is between Q and S, $PR = QS$.
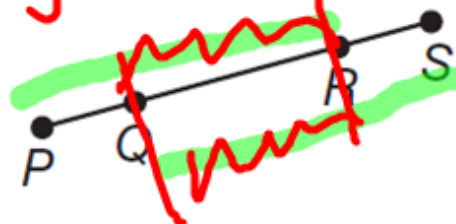
Prove: $PQ = RS$

Proof:

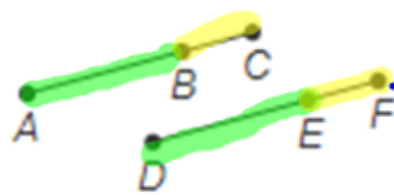| Statements	Reasons
1. Q is between P and R.	1. Given
2. $PQ + QR = PR$	2. Segment Add Post.
3. R is between Q and S.	3. Given
4. $QR + RS = QS$	4. Seg. Add. Post.
5. $PR = QS$	5. Given
6. $PQ + QR = QR + RS$	6. Subst.
7. $PQ + QR - QR = QR + RS - QR$	7. Subt
8. $PQ = RS$	8. Substitution

| Example | Write a two-column proof. |

Given: $\overline{AB} \cong \overline{DE}$; $\overline{BC} \cong \overline{EF}$

Prove: $\overline{AC} \cong \overline{DF}$

Proof:

Statements	Reasons
1. $\overline{AB} \cong \overline{DE}$	1. ⎫ Given
2. $\overline{BC} \cong \overline{EF}$	2. ⎭
3. $AB = DE$	3. ⎫ Dfn of —
4. $BC = EF$	4. ⎭
5. $AB + BC = DE + EF$	5. Add Prop
6. $AB + BC = AC$, $DE + EF = DF$	6. Seg Add Prop
7. $AC = DF$	7. Subst.
8. $\overline{AC} \cong \overline{DF}$	8. Dfn of $\cong$

Justify each statement with a property of congruence.

1. If $\overline{DE} \cong \overline{GH}$, then $\overline{GH} \cong \overline{DE}$.   Symm.

2. If $\overline{AB} \cong \overline{RS}$ and $\overline{RS} \cong \overline{WY}$ then $\overline{AB} \cong \overline{WY}$.   trans.
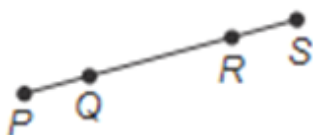
3. $\overline{RS} \cong \overline{RS}$ Reflexive

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4. Complete the proof.

Given: $\overline{PR} \cong \overline{QS}$

*Prove: $\overline{PQ} \cong \overline{RS}$

Proof:

Statements	Reasons
a. $\overline{PR} \cong \overline{QS}$	a. Given
* b. $PR = QS$	b. Dfn of $\cong$
c. $PQ + QR = PR$	c. Seg Add Post
d. $QR + RS = QS$	d. Segment Addition Postulate
→ e. $PQ + QR = QR + RS$	e. Subst.
f. $PQ = RS$	f. Subtraction Property
g. $\overline{PQ} \cong \overline{RS}$	g. Definition of congruence of segments